# Worksheet for Sections 9.8 and 9.9

1. (a) Define carefully: "The series $\sum_{n=0}^{\infty} b_n$ converges absolutely," and write down a series that converges absolutely, and one that converges conditionally.

   (b) Define the radius of convergence $R$ of a given power series, and write down a power series whose radius of convergence is $\sqrt{3}$.

   (c) Tell whether the interval of convergence $I$ of a power series necessarily stays the same when the power series is differentiated. Explain your answer by quoting an appropriate theorem.

   (d) Find the radius of convergence of the power series for

   (a) $e^{3x}$, (b) $\sin(2x)$, (c) $\dfrac{2}{1-x^2}$.

2. Let $f(x) = \cos(2x)$. This problem focuses on the Taylor series of $f$ about 0.

   (a) In the middle of p. 640 there is a formula for the Taylor series of $\cos x$ about 0. Use it to find the Taylor series of $\cos(2x)$ about 0.

   (b) Write down the 26th and the 27th Taylor polynomials of $f$. Are they the same? Explain your answer.

   (c) Show that $|f^{(n+1)}(x)| \le 2^{n+1}$ for all $x$ and all positive integers $n$, and then use the Lagrange Remainder Formula ((11) in Section 9.9) to show that $\lim_{n\to\infty} r_n(x) = 0$ for all $x$. (*Hint*: You will need to use Corollary 9.21 for the limit.)

3. Suppose we were to plot (about 0) the 1000th Taylor polynomial $p_{1000}$ of $\sin x$.

   (a) What do you expect would be the relationship between the graphs of $p_{1000}$ and $\sin x$? Explain. (You might study Figure 9.23 in the book before answering.)

   (b) Find $\lim_{x\to\infty} p_{1000}(x)$. What can you say about $|p_{1000}(x) - \sin x|$ for large values of $x$?

   (c) Let $R > 0$. Use the Lagrange Remainder Formula to determine an upper bound for the nth Taylor remainder $r_n(x)$ for all $x$ in $[-R, R]$.

   (d) What conclusion can you draw from (b) and (c)?